Math 55 Discussion problems 31 Jan

1. Show that if a, b, and c are real numbers and $a \neq 0$, then there is a unique solution of the equation $ax + b = c$.

2. Suppose that five ones and four zeros are arranged around a circle. Between any two equal bits you insert a 0 and between any two unequal bits you insert a 1 to produce nine new bits. Then you erase the nine original bits. Show that when you iterate this procedure, you can never get nine zeros. [Hint: Work backward, assuming that you did end up with nine zeros.]

3. Prove that there are no solutions in integers x and y to the equation $2x^2 + 5y^2 = 14$.

4. Prove that between every rational number and every irrational number there is an irrational number.

5. Disprove the statement that every positive integer is the sum of at most two squares and a cube of nonnegative integers.